Problem Set I: Due Tuesday, January 22, 2013
Should be completed by January 17, 2013

1.) Calculate Q, the rate of collisionless power dissipation, for the electrostatic electron plasma (Langmuir) wave with $\omega = \omega(k)$. You may assume 1D, and a Maxwellian $\langle f \rangle$. Calculate $\omega(k)$ explicitly.

Note: Recall $Q = \omega(k) \in_{IM} \frac{|E_k|^2}{8\pi}$, where $\in_{IM} = \in_{IM}(k, \omega(k))$.

2.) Consider a current-driven system in 1D. Electrons are Maxwellian, with centroid at u_0, temperature T_e. Ions have temperature T_i. Assume the system is collisionless.

(a.) Derive an expression for the mean electric field required to maintain the electron mean velocity u_0. Your answer should depend on $\tilde{E}$ and $\tilde{f}$.

(b.) Calculate the general condition for stability of this system. Do not assume $k^2 \lambda_D^2 << 1$. Be as explicit as possible. (You can ignore the external field here.)

(c.) Assuming a spectrum of unstable CDIA waves, derive an expression for the mean electric field required to maintain a stationary state. You should leave your answer as a function of u_0 and the wave spectrum.

(d.) Prove that total resonant particle energy and total wave energy are conserved here, at the level of quasi-linear theory. N.B. You must consider both resonant electrons and resonant ions.

3.) Consider a 1D system of a cold beam, with density n_0 and velocity V_0 interacting with a cold plasma of density n_0.

(a.) Calculate the wave energy of modes of the beam only. Explain your results.

(b.) Calculate the condition for instability, assuming $n_b = n_0$. Explain the physics of your result. Hint: Choose your frame carefully!

(c.) Now take $n_b << n_0$. Estimate when instability occurs. Explain the physics of your result.

4.) *Electron MHD* (EMHD)

This extended problem introduces you to EMHD and challenges you to apply what you've learned about MHD to understand the structures of a different system of fluid equations. In EMHD, the ions are stationary and the "fluid" is a fluid of electrons. EMHD is useful in problems involving fast Z-pinches, filamentation and magnetic field generation in laser plasmas, Fast Igniter, etc.

The basic equations of EMHD are the electron momentum balance equation

(1) $$\frac{\partial}{\partial t}\underline{v} + \underline{v}\cdot\underline{\nabla}\underline{v} = -\frac{q}{m}\underline{E} - \frac{\nabla P}{\rho} - \frac{q}{mc}\left(\underline{v}\times\underline{B}\right) - \nu\underline{v},$$

(2) $$\underline{J} = -nq\underline{v},$$

and continuity

(3) $$\underline{\nabla}\cdot\underline{J} = 0.$$

Note that here, Ampere's law forces incompressibility of the mass flow $\rho\underline{v}$. Here $\underline{v}$ is the electron fluid velocity, ν is the electron-ion collision frequency, $q = |e|$, $m = m_e$. Of course, Maxwell's equations apply, but the displacement current is neglected.

i.) *Freezing-in*

Determine the freezing-in law for EMHD by taking the curl of Eqn. (1) and using the identity

$$\underline{\mathrm{v}} \cdot \underline{\nabla}\underline{\mathrm{v}} = \underline{\mathrm{v}} \times \underline{\omega} - \underline{\nabla}\left(\mathrm{v}^2/2\right).$$

Assume the electrons have $p = p(\rho)$. Approach this problem by trying to derive an equation for "something" which has the structure of the induction equation in MHD. Discuss the physics - what is the "something" and what is it frozen into? In retrospect, why is the frozen-in quantity obvious? How is freezing-in broken?

ii.) *Large Scale Limit*

Show that for $\ell^2 >> c^2/\omega_{pe}^2$, the dynamical equations for EMHD reduce to

$$\frac{\partial B}{\partial t} + \underline{\nabla} \times \left(\frac{\underline{J}}{nq} \times \underline{B}\right) = -\nu \underline{\nabla} \times \left(\frac{\underline{J}}{nq}\right)$$

$$\underline{\nabla} \cdot \underline{J} = 0; \quad \underline{\nabla} \cdot \underline{B} = 0.$$

a.) Show that density remains constant here.

b.) Formulate an energy theorem for EMHD in this limit, by considering the energy content of a "blob" of EMHD fluid.

c.) Discuss the frozen-in law in this limit.

5.) Kulsrud, Chapter 3, #1

6.) Kulsrud, Chapter 3, #3

7.) Kulsrud, Chapter 3, #4

8.) Kulsrud, Chapter 4, #1, paragraph 1

9.) Kulsrud, Chapter 4, #2

10.) Kulsrud, Chapter 4, #4